U6 Chapter 4

Binomial Expansion

Chapter Overview

1. Binomial Series Recap

2. Binomial Expansion for negative/fractional powers

3. Constant is not 1: $\left(a+b\right)^{n}$

4. Using Partial Fractions



The Binomial Series: Recap

Recall that if n is a positive integer

$$\left(a+b\right)^{n}=a^{n}+nC1 a^{n-1}b+nC2 a^{n-2}b^{2}+…$$

$$\left(1+x\right)^{n}=1+nx+\frac{n\left(n-1\right)}{2!}x^{2}+\frac{n\left(n-1\right)\left(n-2\right)}{3!}x^{3}+…+nCr x^{n}$$

Also $\left(a+b\right)^{n}=a^{n}\left(1+\frac{b}{a}\right)^{n}$

Examples

1. Expand $\left(1+x\right)^{11}$ up to and including the term in $x^{3}$

2. Expand $\left(1-2x\right)^{8} $ up to and including the term in $x^{3}$

Binomial Expansion for Negative/ Fractional Powers

Example

1. Use the binomial expansion to find the first four terms of $\frac{1}{1+x}$

2. Use the binomial expansion to find the first four terms of $\sqrt{1-3x}$

An infinite expansion $\left(1+x\right)^{n}$ is valid if $\left|x\right|<1$

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Quickfire Examples:

1. Expansion of $\left(1+2x\right)^{-1}$ valid if:

2. Expansion of $\left(1-x\right)^{-2}$ valid if:

3. Expansion of $\left(1+\frac{1}{4}x\right)^{\frac{1}{2}}$ valid if:

4. Expansion of $\left(1-\frac{2}{3}x\right)^{-1}$ valid if:

Combining Expansions

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Test Your Understanding

1. Find the binomial expansion of $\frac{1}{\left(1+4x\right)^{2}}$ up to an including the term in $x^{3}$. State the values of $x$ for which the expansion is valid.

2.



Extension

[STEP I 2011 Q6] Use the binomial expansion to show that the coefficient of $x^{r}$ in the expansion of $\left(1-x\right)^{-3}$ is $\frac{1}{2}\left(r+1\right)\left(r+2\right)$.

1. Show that the coefficient of $x^{r}$ in the expansion of $\frac{1-x+2x^{2}}{\left(1-x\right)^{3}}$ is $r^{2}+1$ and hence find the sum of the series
$$1+\frac{2}{2}+\frac{5}{4}+\frac{10}{8}+\frac{17}{16}+\frac{26}{32}+\frac{37}{64}+…$$
2. Find the sum of the series
$$1+2+\frac{9}{4}+2+\frac{25}{16}+\frac{9}{8}+\frac{49}{64}$$

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